

# Fragmentation Contributions to $J/\psi$ Production at the Tevatron and the LHC

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We compute leading-power fragmentation corrections to  $J/\psi$  production at the Tevatron and the LHC. We find that, when these corrections are combined with perturbative corrections through next-to-leading order in the strong coupling constant  $\alpha_s$ , we obtain a good fit to high- $p_T$  cross section data from the CDF and CMS Collaborations. The fitted long-distance matrix elements lead to predictions of near-zero  $J/\psi$  polarization in the helicity frame at large  $p_T$ .

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Much of the current phenomenology of heavy-quarkonium production in high-energy collisions is based on the effective field theory nonrelativistic QCD (NRQCD) [1]. Specifically, calculations are based on the NRQCD factorization conjecture [2], which states that the inclusive cross section to produce a quarkonium state  $H$  at large momentum transfer in a collision of particles  $A$  and  $B$  can be expressed as

$$d\sigma_{A+B \rightarrow H+X} = \sum_n d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n) \rangle. \quad (1)$$

Here, the  $d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X}$  are perturbatively calculable short-distance coefficients (SDCs), which are, essentially, the partonic cross sections to produce a heavy-quark-antiquark pair  $Q\bar{Q}(n)$  in a particular color and angular-momentum state  $n$ , convolved with parton distributions. The  $\langle \mathcal{O}^H(n) \rangle$  are nonperturbative, long-distance matrix elements (LDMEs) of NRQCD operators and are, essentially, the probabilities for the pair  $Q\bar{Q}(n)$  to evolve into a quarkonium state  $H$  plus anything. The LDMEs are conjectured to be universal, *i.e.*, process independent. This conjecture implies that information that is gained about the LDMEs by studying one quarkonium production process can be used to make predictions about another.

The LDMEs have a well-defined scaling with the relative velocity  $v$  of the  $Q$  and the  $\bar{Q}$  in the quarkonium center-of-momentum frame. Consequently, the sum over  $n$  in Eq. (1) is actually an expansion in powers of  $v$ , where  $v^2 \approx 0.25$  for the  $J/\psi$  charm-anticharm ( $c\bar{c}$ ) state. In present-day quarkonium-production phenomenology, the sum over  $n$  is usually truncated at relative order  $v^4$ . Four  $Q\bar{Q}$  states appear in this truncation:  $Q\bar{Q}(^3S_1^{[1]})$ ,  $Q\bar{Q}(^1S_0^{[8]})$ ,  $Q\bar{Q}(^3S_1^{[8]})$ , and  $Q\bar{Q}(^3P_J^{[8]})$ , where we use standard spectroscopic notation for the angular momentum, and the superscripts 1 and 8 denote color-singlet and color-octet states, respectively.

Three groups have now completed the formidable task of calculating the SDCs that appear in the four-LDME truncation through next-to-leading order (NLO) in the

QCD coupling  $\alpha_s$  [3–9]. Generally, the NLO calculations, combined with the four-LDME phenomenology, lead to reasonable agreement with a wide range of inclusive  $J/\psi$  production measurements that have been made at the Tevatron, the LHC, and the  $B$  factories [10, 11]. Problematic exceptions to this agreement arise from NLO predictions, which are based on fits to  $J/\psi$  cross sections, that the  $J/\psi$  polarization in the helicity frame is substantially transverse at large  $J/\psi$  transverse momentum  $p_T$  [9, 11, 12]. Measurements of the  $J/\psi$  polarization at the Tevatron [13, 14] and the LHC [15, 16] are in contradiction with these predictions [17].

In this Letter we make use of the leading-power (LP) factorization formalism to compute fragmentation contributions to  $J/\psi$  production beyond NLO that appear in the leading power of  $m_{J/\psi}^2/p_T^2$  for  $p_T \gg m_{J/\psi} \approx 2m_c$ , where  $m_{J/\psi}$  is the  $J/\psi$  mass and  $m_c$  is the charm-quark mass. Specifically, we include contributions that arise from parton production cross sections (PPCSs), computed through order  $\alpha_s^3$  (NLO), convolved with fragmentation functions (FFs) for a single parton to fragment into a  $J/\psi$ , computed through order  $\alpha_s^2$  and computed to all orders in  $\alpha_s$  for leading logarithms of  $p_T^2/(2m_c)^2$ . This procedure reproduces only part of the full next-to-next-to-leading-order NRQCD contribution in the large- $p_T$  limit. However, within the LP factorization framework, it is consistent to calculate the PPCSs and the FFs separately to a given accuracy, as we do here. We neglect contributions from the  $^3S_1^{[1]}$  channel, since they have been found to be small through NLO in  $\alpha_s$  [3–9]. We find that the new LP fragmentation contributions that we compute have important effects on the shapes of the SDCs as functions of  $p_T$  and on the relative contributions of the various angular-momentum channels in fits to the experimental  $p_T$  spectra. We are able to obtain good fits to the data of the CDF [19] and CMS [20] Collaborations for prompt  $J/\psi$  production for  $p_T \geq 10$  GeV. The resulting LDMEs lead to a prediction that the  $J/\psi$

polarization in the helicity frame in direct production at both the Tevatron and the LHC is near zero at high  $p_T$ , in good agreement with the CMS data [15] and in greatly improved agreement with the CDF data [13, 14].

LP factorization states that the contribution to the inclusive cross section to produce a hadron  $H$  at LP in  $1/p_T^2$  ( $d\sigma/dp_T^2 \sim 1/p_T^4$ ) can be written as

$$d\sigma_{A+B \rightarrow H+X}^{\text{LP}} = \sum_i d\hat{\sigma}_{A+B \rightarrow i+X} \otimes D_{i \rightarrow H}, \quad (2)$$

where the  $d\hat{\sigma}_{A+B \rightarrow i+X}$  are inclusive PPCSs to produce a parton  $i$ , the  $D_{i \rightarrow H}$  are FFs [21] for the parton  $i$  to fragment into the hadron  $H$ , and  $\otimes$  denotes a convolution with respect to the longitudinal momentum fraction  $z$  of the hadron relative to the fragmenting parton. The PPCSs can be computed in QCD perturbation theory; the FFs are nonperturbative and must be determined phenomenologically. The LP factorization formula in Eq. (2) was proven for the inclusive production of a light hadron in  $e^+e^-$  annihilation in Ref. [21]. The proof of Eq. (2) for production of a heavy quarkonium was sketched in Ref. [22]. For  $J/\psi$  production, the corrections to Eq. (2) are of relative order  $m_c^2/p_T^2$ . Expressions for next-to-leading-power (NLP) factorization for quarkonium production have been derived in Refs. [23, 24].

One can apply LP factorization to the SDCs in Eq. (1). The result is

$$d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X}^{\text{LP}} = \sum_i d\hat{\sigma}_{A+B \rightarrow i+X} \otimes D_{i \rightarrow Q\bar{Q}(n)}, \quad (3)$$

where both the PPCSs  $d\hat{\sigma}_{A+B \rightarrow i+X}$  and the FFs  $D_{i \rightarrow Q\bar{Q}(n)}$  can be calculated in QCD perturbation theory. In this Letter, we use the LP factorization approximation (3) for the SDCs to compute contributions that augment the NLO calculations of the SDCs. As we have mentioned, we compute the PPCSs  $d\hat{\sigma}_{A+B \rightarrow i+X}$  through order  $\alpha_s^3$  (NLO), and we compute the FFs  $D_{i \rightarrow Q\bar{Q}(n)}$  through order  $\alpha_s^2$  and, for leading logarithms of  $p_T^2/(2m_c)^2$ , to all orders in  $\alpha_s$ .

Formulas for the PPCSs through order  $\alpha_s^3$  were given in Refs. [25, 26]. We evaluate them by making use of a computer code that was provided by the authors of Ref. [25]. The gluon FFs  $D_{g \rightarrow Q\bar{Q}(n)}$  in Eq. (3) are given for the  $^1S_0^{[8]}$  channel at order  $\alpha_s^2$  (LO) in Refs. [27, 28], for the  $^3S_1^{[8]}$  channel at order  $\alpha_s$  (LO) in Ref. [29] and at order  $\alpha_s^2$  (NLO) in Refs. [30, 31], and for the  $^3P_J^{[8]}$  channels at order  $\alpha_s^2$  (LO) in Refs. [28, 29]. The light-quark FF  $D_{q \rightarrow Q\bar{Q}(n)}$  in the  $^3S_1^{[8]}$  channel is given at order  $\alpha_s^2$  (LO) in Ref. [32]. Light-quark fragmentation in the other color-octet channels vanishes through order  $\alpha_s^2$ .

We calculate, to all orders in  $\alpha_s$ , contributions to the FFs from leading logarithms of  $p_T^2/(2m_c)^2$  by making use of the LO Dokshitzer-Gribov-Lipatov-Altarelli-Parisi

(DGLAP) evolution equation [33–36]:

$$\frac{d}{d \log \mu_f^2} \begin{pmatrix} D_S \\ D_g \end{pmatrix} = \frac{\alpha_s(\mu_f)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{gq} \\ P_{qg} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} D_S \\ D_g \end{pmatrix}, \quad (4)$$

where  $D_g = D_{g \rightarrow Q\bar{Q}(n)}$ ,  $D_S = \sum_f [D_{q_f \rightarrow Q\bar{Q}(n)} + D_{\bar{q}_f \rightarrow Q\bar{Q}(n)}]$ ,  $f$  is the light-quark or light-antiquark flavor,  $n_f = 3$  is the number of active quark flavors, the  $P_{ij}$ 's are the splitting functions for the FFs, and  $\mu_f$  is the factorization scale. We have suppressed the dependence of  $D_{i \rightarrow Q\bar{Q}(n)}$  and  $d\hat{\sigma}_{A+B \rightarrow i+X}$  on  $\mu_f$ . We solve Eq. (4) by taking a Mellin transform with respect to  $z$ , integrating  $\mu_f$  from  $2m_c$  to  $m_T \equiv \sqrt{p_T^2 + 4m_c^2}$  in order to incorporate the logarithms of  $m_T^2/(2m_c)^2 \approx p_T^2/(2m_c)^2$ , and taking an inverse Mellin transform in order to obtain a  $z$ -space expression.

In order to avoid double counting, we must subtract from  $d\sigma_{A+B \rightarrow Q\bar{Q}(n)+X}^{\text{LP}}$  [Eq. (3)] contributions through order  $\alpha_s^3$ , which also appear in the LO and NLO calculations of the SDCs. We denote these contributions by  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$ , and we denote the sum of the LO and NLO contributions to the SDCs by  $d\sigma_{\text{NLO}}/dp_T$ . The contributions of  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$  to  $J/\psi$  production at the LHC at the center-of-momentum energy  $\sqrt{s} = 7$  TeV are compared with  $d\sigma_{\text{NLO}}/dp_T$  in Fig. 1. We have taken  $d\sigma_{\text{NLO}}/dp_T$  from the calculation of Refs. [7, 8]. In order to maintain compatibility with that calculation, we have taken  $m_c = 1.5 \pm 0.1$  GeV; used the CTEQ6M parton distributions [37] and the two-loop expression for  $\alpha_s$ , with  $n_f = 5$  quark flavors and  $\Lambda_{\text{QCD}}^{(5)} = 226$  MeV; set the renormalization, factorization, and the NRQCD scales to  $\mu_r = m_T$ ,  $\mu_f = m_T$ , and  $\mu_\Lambda = m_c$ , respectively; and dropped contributions involving more than one heavy-quark-antiquark pair in the final state.

As can be seen from Fig. 1, in the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  channels,  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$  accounts well for the full fixed-order cross section  $d\sigma_{\text{NLO}}/dp_T$  for  $p_T$  greater than 10–20 GeV. However, in the  $^1S_0^{[8]}$  channel,  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$  does not approach  $d\sigma_{\text{NLO}}/dp_T$  until much larger values of  $p_T$ . This is a consequence of the fact that the LP FFs in the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  channels contain  $\delta$  functions and plus distributions (remnants of canceled infrared divergences) that are strongly peaked near  $z = 1$ , while the LP FF in the  $^1S_0^{[8]}$  channel contains no such peaking [28]. The NLO correction in the  $^3S_1^{[8]}$  channel is small relative to the LO contribution because of a cancellation between the NLO parton-scattering contribution and the NLO FF contribution, which contribute about  $-50\%$  and  $+100\%$ , respectively, relative to the LO contribution at  $p_T = 52.7$  GeV.

Our result for the LO plus NLO PPCSs, augmented by the LP fragmentation contributions that we have computed, is given by

$$\frac{d\sigma^{\text{LP+NLO}}}{dp_T} = \frac{d\sigma^{\text{LP}}}{dp_T} - \frac{d\sigma_{\text{NLO}}^{\text{LP}}}{dp_T} + \frac{d\sigma_{\text{NLO}}}{dp_T}, \quad (5)$$

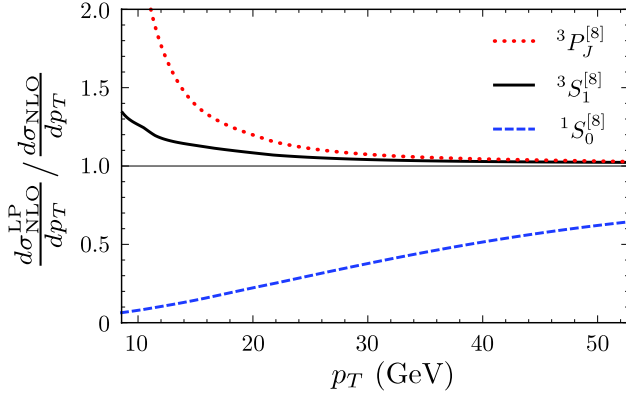


FIG. 1: The ratio  $(d\sigma_{\text{NLO}}^{\text{LP}}/dp_T)/(d\sigma_{\text{NLO}}/dp_T)$  for the  $1S_0^{[8]}$ ,  $3P_J^{[8]}$ , and  $3S_1^{[8]}$  channels in  $pp \rightarrow J/\psi + X$  at  $\sqrt{s} = 7$  TeV.

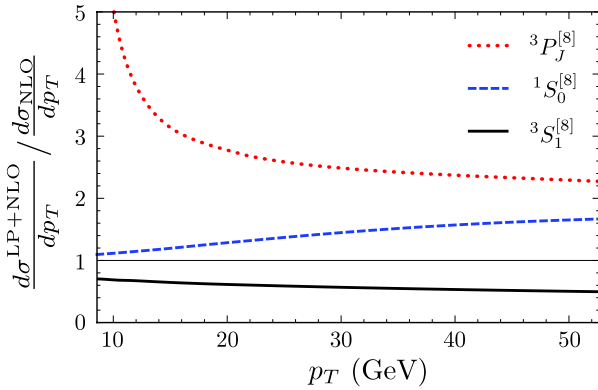


FIG. 2: The ratio  $(d\sigma^{\text{LP+NLO}}/dp_T)/(d\sigma_{\text{NLO}}/dp_T)$  for the  $1S_0^{[8]}$ ,  $3P_J^{[8]}$ , and  $3S_1^{[8]}$  channels in  $pp \rightarrow J/\psi + X$  at  $\sqrt{s} = 7$  TeV.

where  $d\sigma^{\text{LP}}/dp_T$  is the LP fragmentation contribution computed to the accuracy described above. In Fig. 2, we compare  $d\sigma^{\text{LP+NLO}}/dp_T$  with  $d\sigma_{\text{NLO}}/dp_T$  in each channel. The LP corrections in the  $3S_1^{[8]}$  and  $1S_0^{[8]}$  channels grow in magnitude with increasing  $p_T$ , reaching  $-50\%$  and  $70\%$ , respectively, at  $p_T = 50$  GeV. The LP corrections are quite dramatic in the  $3P_J^{[8]}$  channel, partly because the LO and NLO contributions tend to cancel at low  $p_T$ .  $d\sigma^{\text{LP+NLO}}/dp_T$  is  $80\%$ – $160\%$  larger than  $d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$  in this channel. These large corrections suggest that the perturbation expansion may be converging slowly.

The contributions that we have calculated are dominated by  $gg$  initial states, which account for about  $70\%$  of the cross section at  $p_T = 52.7$  GeV. Contributions from light-quark fragmentation and from  $q-g$  mixing in the DGLAP equation amount to only about  $1\%$  and less than  $1\%$  of the cross section, respectively, at  $p_T = 52.7$  GeV.

At  $p_T = 52.7$  GeV, the all-orders resummation of leading logarithms contributes about  $-43\%$  in the  $3S_1^{[8]}$  channel relative to the LO fragmentation contribution. Es-

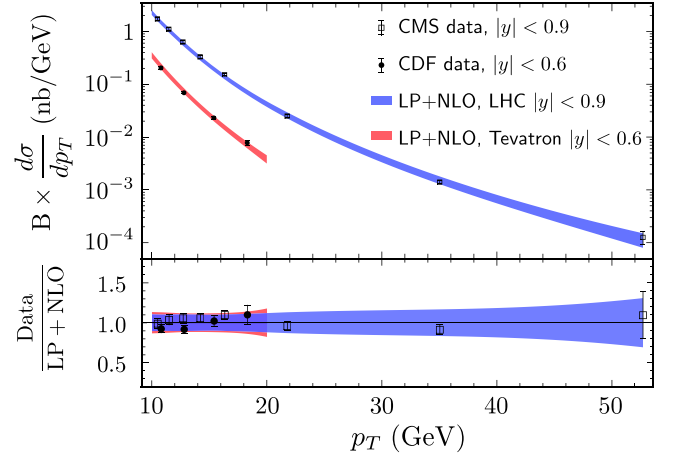


FIG. 3: LP+NLO predictions for the  $J/\psi$  differential cross section at the LHC and Tevatron compared with the CMS [20] and CDF data [19].  $B = 5.93 \times 10^{-2}$  is the branching ratio for  $J/\psi \rightarrow \mu^+ \mu^-$  [38].

entially all of that is already accounted for in the NLO contribution. In the  $1S_0^{[8]}$  and  $3P_J^{[8]}$  channels, the all-orders resummations contribute only  $2\%$  and  $5\%$ , respectively, relative to the NLO fragmentation contribution, owing to an accidental cancellation between contributions from the running of  $\alpha_s$  and contributions from the DGLAP splitting. Hence, in each channel,  $d\sigma^{\text{LP}}/dp_T - d\sigma_{\text{NLO}}^{\text{LP}}/dp_T$  is given to good approximation by the contribution from the NLO PPCSs convolved with the order- $\alpha_s^2$  contribution to the FF.

If we vary  $\mu_r$  and  $\mu_f$  separately between  $2m_T$  and  $m_T/2$ , then half of the difference between the maximum and minimum values of the SDCs is less than about  $25\%$  of the central value over the  $p_T$  range that we consider. These relatively small scale variations suggest that perturbation series may be under reasonable control. Overall factors of  $m_c$  in the SDCs can be absorbed into redefinitions of the LDMEs, and, hence, the uncertainty in  $m_c$  from these factors does not affect fits to the cross sections or the polarization predictions that we make. The residual  $p_T$ -dependent effects from the uncertainty in  $m_c$  are less than about  $5\%$ . Therefore, in fitting the data, we assume that the theoretical uncertainty is  $25\%$ . This value is also typical of the uncertainty that one would expect from corrections of higher order in  $v$ .

In Fig. 3, we show a combined fit of our cross section predictions to CDF [19] and CMS [20] data for prompt  $J/\psi$  production. In obtaining these fits, we have included only data with  $p_T \geq 10$  GeV in order to suppress NLP corrections. The resulting fit is quite good, with  $\chi^2/\text{d.o.f.} = 0.085$ , suggesting that higher-order corrections do not affect the  $p_T$  dependences of the SDCs at the level of our  $25\%$  estimate of the theoretical uncertainty. The fit leads to the following values for the LDMEs:  $\langle \mathcal{O}^{J/\psi}(1S_0^{[8]}) \rangle = 0.099 \pm 0.022 \text{ GeV}^3$ ,

$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle = 0.011 \pm 0.010 \text{ GeV}^3$ , and  $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle = 0.011 \pm 0.010 \text{ GeV}^5$ . The corrections that we have computed result in very similar shapes at large  $p_T$  for the SDCs for the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  channels. As  $p_T$  increases, these SDCs fall much more slowly than do the experimental data. On the other hand, the contribution of the  $^1S_0^{[8]}$  channel, including the corrections that we have computed, matches the shape of the experimental data quite well at large  $p_T$ . Consequently, in fits to the experimental data with  $p_T \geq 10 \text{ GeV}$ , the sum of the contributions of the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  channels tends to be small, with the predominant contribution coming from the  $^1S_0^{[8]}$  channel. While the LDMEs  $\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$  and  $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$  are separately poorly determined, a full covariance analysis shows that the sum of their contributions is constrained to be much less than the  $^1S_0^{[8]}$  contribution. A fit to the CDF and CMS production cross sections that makes use of the NLO SDCs without the LP fragmentation corrections also describes the data well, but does not constrain any of the LDMEs. That fit yields  $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle = -0.030 \pm 0.381 \text{ GeV}^3$ ,  $\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle = 0.023 \pm 0.057 \text{ GeV}^3$ , and  $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle = 0.043 \pm 0.106 \text{ GeV}^5$ , with  $\chi^2/\text{d.o.f.} = 0.239$ .

At high  $p_T$ , the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  contributions are both nearly 100% transversely polarized. Hence, the small size of the sum of the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  contributions implies that the  $J/\psi$ 's are produced largely unpolarized at high  $p_T$ . (This cancellation of the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  polarization contributions was discussed in Ref. [18], in which CDF polarization data were used to constrain the LDME fit.) Assuming that the  $^3S_1^{[8]}$  and  $^3P_J^{[8]}$  contributions are 100% transversely polarized, we obtain the polarization predictions that are shown in Fig. 4. These predictions agree with the CMS [15] and CDF [13, 14] polarization data much better than do the predictions from the NLO calculations in Refs. [9, 11, 12]. The predictions in Refs. [9, 11, 12] rely on data at  $p_T < 10 \text{ GeV}$  to constrain the LDMEs. As we have mentioned, a fit to  $J/\psi$  production cross sections at  $p_T \geq 10 \text{ GeV}$  that makes use of the NLO SDCs does not constrain the LDMEs. Consequently, it does not yield a definite prediction for the polarization.

The LP-fragmentation corrections to  $J/\psi$  production that we have described in this Letter result in substantial changes to the predictions of NRQCD factorization for  $J/\psi$  production. This initial investigation suggests that these corrections might resolve the long-standing conflict between NRQCD factorization predictions for quarkonium polarizations and the polarization measurements that have been made in collider experiments. Several *caveats* should be mentioned. First, we are comparing theoretical predictions for direct  $J/\psi$  production with prompt  $J/\psi$  production data that include feed down from the  $\chi_{cJ}$  and  $\psi(2S)$  states. Collider experiments have yet

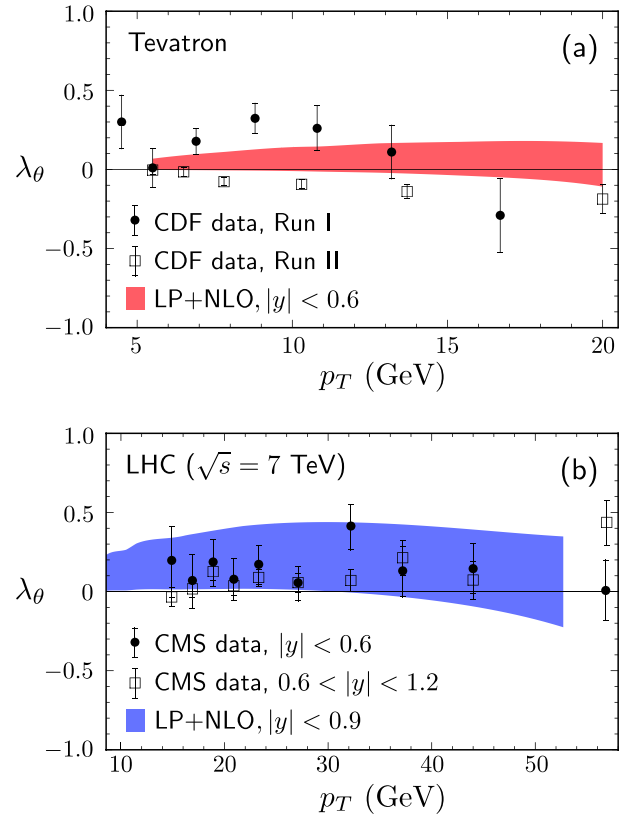


FIG. 4: LP+NLO predictions for the  $J/\psi$  polarization parameter  $\lambda_\theta \equiv (\sigma_T - 2\sigma_L)/(\sigma_T + 2\sigma_L)$  compared with (a) the CDF Run I [13] and CDF Run II [14] data and (b) the CMS [15] data. Here,  $\sigma_T$  ( $\sigma_L$ ) is the cross section for transversely (longitudinally) polarized  $J/\psi$ 's.

to determine whether feed-down effects substantially alter shapes of differential cross sections or measured polarizations. Second, the large sizes of the corrections that arise from the parton-scattering cross sections at NLO suggest that the perturbation expansion may not yet be under good control. Investigations of higher-order corrections to the PPCs and FFs should be pursued, as should NLP fragmentation corrections. Finally, the approach that has been presented in this Letter should be tested for additional quarkonium states, such as the  $\chi_{cJ}$ ,  $\Upsilon(nS)$ , and  $\chi_{bJ}$  states, and for additional production processes.

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